$R_{X,0}$. Calculations and available data show that the number of sodium dust particles required to produce the Spitzer concentration of electrons in the base region could have been present during the First Set of runs.[5]

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Strouhal Numbers for the Hypersonic Wakes of Spheres and Cones

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Introduction

Fay and Goldburg have presented evidence of the fluid mechanical similarity of the unsteady hypersonic wake and the unsteady incompressible wake.[1] The Strouhal number ($S = fL/V$ where V is the freestream velocity, f is the wake periodic frequency, and L is a characteristic length) is a characterization of the prominent large scale (wavelength) frequency in the unsteady wake. Recently it was found that the Strouhal number variation for the incompressible wake for a range of cones and spheres could be correlated using a Reynolds number based on total wake momentum thickness as the characteristic length.[2] This paper presents a similar finding for the unsteady hypersonic wake.

Goldburg and Florsheim[2] investigated the unsteady incompressible wake behind various three-dimensional shapes by dropping small bodies into a tank full of an aqueous glycerin solution and observing their wakes. The wakes were rendered visible by a dye from the body as it passed through the fluid. Equation (1) defines the total wake momentum thickness θ:

$$\rho V^2 \pi^j \theta^{j+1} = \text{drag} = \tfrac{1}{2}\rho V^2 C_D A \qquad \begin{cases} j = 0 \text{ planar} \\ j = 1 \text{ axisymmetric} \end{cases} \tag{1}$$

If one views the body as a black box of arbitrary shape leaving a given wake, and one asks what length parameter best characterizes the shear flow wake phenomenon produced by the dissipation processes connected with the body, then the answer appears to be the total wake momentum thickness θ or equivalently from Eq. (1), $(C_D A/2\pi)^{1/2}$. The incompressible results of Goldburg and Florsheim tend to support the suggestion that total wake momentum thickness

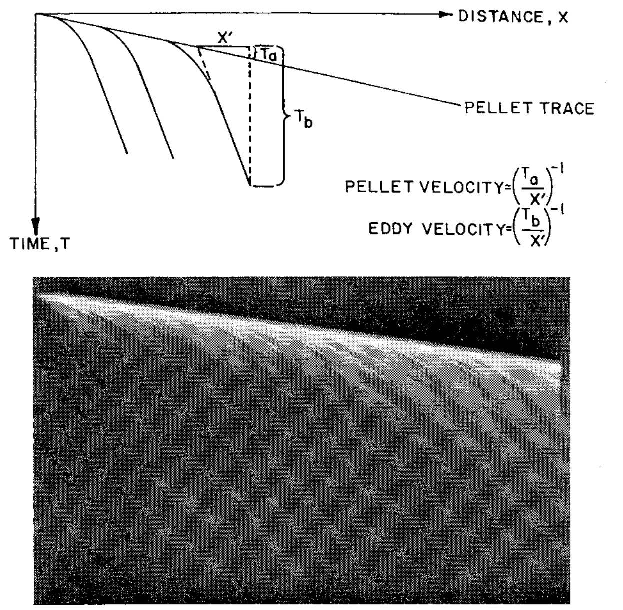

Fig. 1 Typical drum camera streak photograph with the corresponding space-time plot. Run: 10-4-61/2; 6 cm. air; 14,700 fps; 0.22-in. diam., and 15-in. field.

is the appropriate length for the variation of Strouhal number with Reynolds number. It was found that for regular vortex shedding, the data for spheres and a range of cones could be correlated with the Rayleigh Strouhal formula based on θ for the incompressible wake:

$$S_\theta = 0.305[1 - (95/Re_\theta)] \tag{2}$$

The disk and the needle were deviate cases.[2]

Investigation

Measurements of the frequency of vortex generation behind hypervelocity spheres in air ($M_\infty \simeq 14$) were made from self-luminous drum camera photographs taken in the Avco-Everett Research Laboratory (AERL) ballistic range and in the ballistic range of the Canadian Armament Research and Development Establishment (CARDE) at Valcartier, Quebec.[1] Measurements behind hypervelocity cones at approximately the same Mach number were also made at the CARDE range. In the photographic techniques used, a streak drum camera records the flow field made

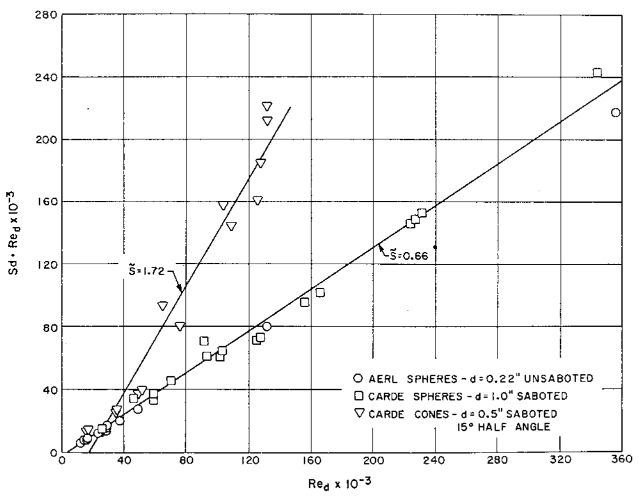

Fig. 2 Strouhal numbers for the hypersonic wake ($M_\infty \approx 14$), spheres and cones; based on body diameter as characteristic length.

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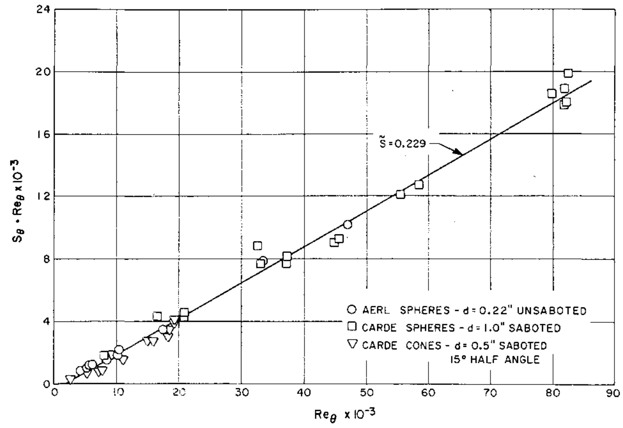

Fig. 3 Strouhal numbers for the hypersonic wake ($M_\infty \approx$ 14), spheres and cones; based on total wake-momentum thickness $\theta = (C_D A/2\pi)^{1/2}$ as characteristic length.

visible by the wake's own luminosity. As the model and its wake move down the ballistic range their image moves transverse to the motion of the film producing on the film an *x-t* diagram of the flow field events. Figure 1 is a typical example of a drum camera streak photograph. The pellet is moving from left to right and traces the bright straight line; the "streaks" are the gross traces of the large scale eddies as they fall behind the body and slow down. The hypersonic Strouhal number is obtained by eye by counting the number of prominences along a line of the film parallel to the trace of the pellet. A counting line is chosen as near to the body as the streak structure is resolvable, i.e., immediately downstream of the base resolution limit of the drum camera technique. The following rules are observed: 1) each prominence is counted and is associated with one cycle of the periodic phenomenon, and 2) fluctuations within an eddy or with a wavelength less than a body radius are ignored. If the projectile moves a real distance x in the time during which N events are counted, then the Strouhal number is NL/x.

A shoulder Reynolds number (flow direction parallel to freestream) is computed as being representative of the separated flow conditions in the near wake where the measurement is made. The best estimates of the flow chemistry indicate an equilibrium flow across the sphere bow shock for the ballistic range conditions of these runs. As in Fay and Goldburg, the formula for the shoulder Reynolds number of spheres in air is taken approximately as

$$R = 1780[V(\text{kft/sec})]^{0.6}[\rho(\text{cm})][L(\text{cm})] \qquad (3)$$

A shoulder Reynolds number for cones is computed by taking the flow through the shock wave and the Prandtl-Meyer expansion of the cone angle using the conical flow tables of Romig[3] and NACA 1135.[4]

Results

Rayleigh's empirical relationship between the Strouhal and Reynolds number

$$S = \tilde{S}\,[1 - (Re_T/Re)]$$

(where $\tilde{S}$ is the asymptotic value of the Strouhal number for large Re, and Re_T is the Reynolds number for which S goes to zero) yields a straight line when $S \times Re$ is plotted against Re. This line when extended to $S = 0$ will yield a transition Reynolds number Re_T.

Figure 2 is such a plot of $S \times Re$ vs Re based on body diameter as the characteristic length. Indicated on this plot are two lines

$$S_d = 0.66(1 - 3180/Re_d) \qquad \text{spheres} \qquad (4)$$

$$S_d = 1.72(1 - 18100/Re_d) \qquad \text{cones} \qquad (5)$$

Each line is the least-square fitted line passing through the respective data. Figure 3 shows the same data plotted using θ as the characteristic length. Based on the preceding arguments, as in the incompressible case, we expect that θ may be the correlating length parameter allowing a single line to be drawn through both the sphere and cone points. In the plot is shown the least-square fitted line

$$S_\theta = 0.229[1 - (1820/Re_\theta)] \qquad (6)$$

Finally, since the frequency measurements are based on highlights of luminosity in the hypersonic wake, the question is raised whether the apparent periodicity is caused by unsymmetrical heating of the projectile in the gun barrel. The answer is apparently not. Calculations show that the mechanical integrity of the bodies would be jeopardized at the kilocycle frequencies involved; in addition, the spheres shot at the AERL range were unsaboted whereas those at CARDE were saboted and thus protected from gun-barrel heating.

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Forced Vibrations of a Burning Rocket

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Introduction

THE loss of mass and stiffness of a burning cylindrical grain affects the dynamic response of a solid-propellant rocket. The viscoelastic behavior of the solid-propellant material is another factor that influences the forced vibrations of a burning rocket. In the present note the effects of ablation and viscoelastic damping are considered in a study of the dynamic response of an encased viscoelastic cylinder with an ablating inner surface.

A time-dependent pressure is applied at the ablating inner surface of the cylinder. The cylinder material is viscoelastic in shear, and it is assumed incompressible in bulk. As a consequence of the incompressibility assumption the dilatational wave velocity is infinite, and a forced vibration is immediately started without initial wave effects. Solid-propellant materials show very high bulk moduli, and they are often considered as incompressible.

Special attention has been devoted to the circumferential stress at the ablating inner surface. The analysis is valid for arbitrary ablation rates. The analogous problem of the encased elastic cylinder has been discussed in an earlier note.[1]

Statement of the Problem

A long viscoelastic cylinder is considered with a circular port of monotonically increasing radius $a(t)$ and a constant

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